APPENDIX

American Monte Carlo
Longstaff-Schwartz Technique

American Monte Carlo for Convertibles
Monte Carlo Technique

- The value of the derivative security can be defined as a risk neutral expectation of a discounted payoff.

$$P = e^{-rT} E\left[P_T \right]$$

or

$$P = e^{-rT} \int_0^{\infty} f(S) P(S) dS$$

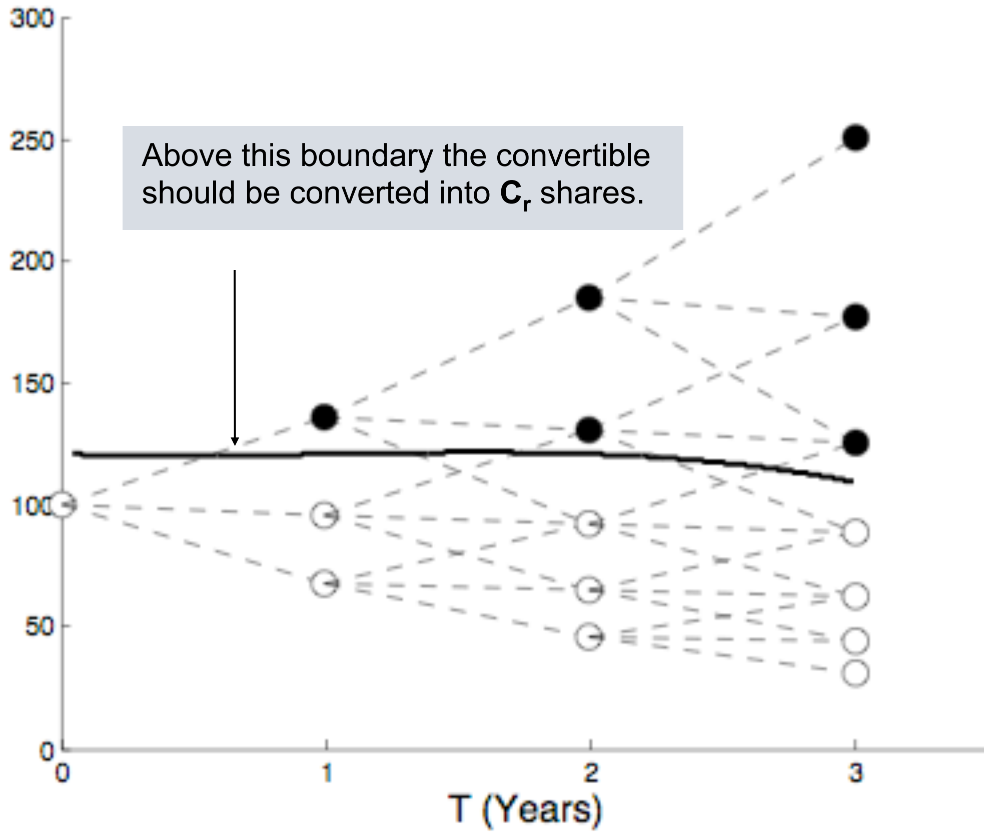
- **Procedure**
  1. Generate N random prices S_i with i=1,...n at maturity T.
  2. Calculate for each S_i the corresponding payoff P_i using the appropriate payoff function $P_{\{t=T\}}$. For derivatives where the final payoff $P_{\{t=T\}}$ depends on a set of m share prices observed at intermediate dates t_k with k=1,...m, will need a slightly different approach. In this step the final payoff $P_{\{t=T\}}$ is now calculated based on a payoff function $P_{\{t=T\}}$ $(S_1,...S_m)$ where all the intermediate price observations intervene.
  3. Discount each of the P_i at the risk free rate r from T to the valuation date of the option. This gives a discounted payoff $P_{i,0}$.
  – Take the average of these discounted values $P_{i,0}$. to obtain the Monte Carlo estimate.

American Monte Carlo
Longstaff and Schwartz

Appendix 1

American Monte Carlo for Convertibles
Monte Carlo Technique

- Convertibles bonds have an expected life < T
- The moment the convertible is converted, called or put is the stopping time τ
- This forms the exercise boundary b(S,t)
- Can be visualized in a tree model :



$P^* > P^c$

with

$P^* = C_r S$

and

$P^c = \text{Continuation Value}$

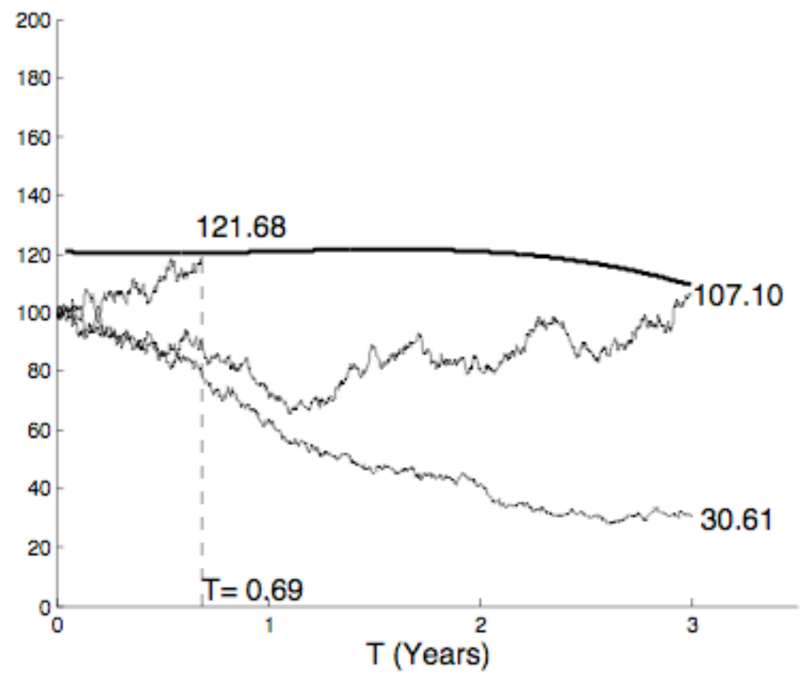
American Monte Carlo for Convertibles Example

• C_r	=	1
• T	=	3 yr
• DivYld	=	5%
• Interest Rate	=	3%
• Volatility (σ)	=	20%
• Face Value	=	100
• S	=	100

- No Call
- No Put

American Monte Carlo for Convertibles
Monte Carlo Technique

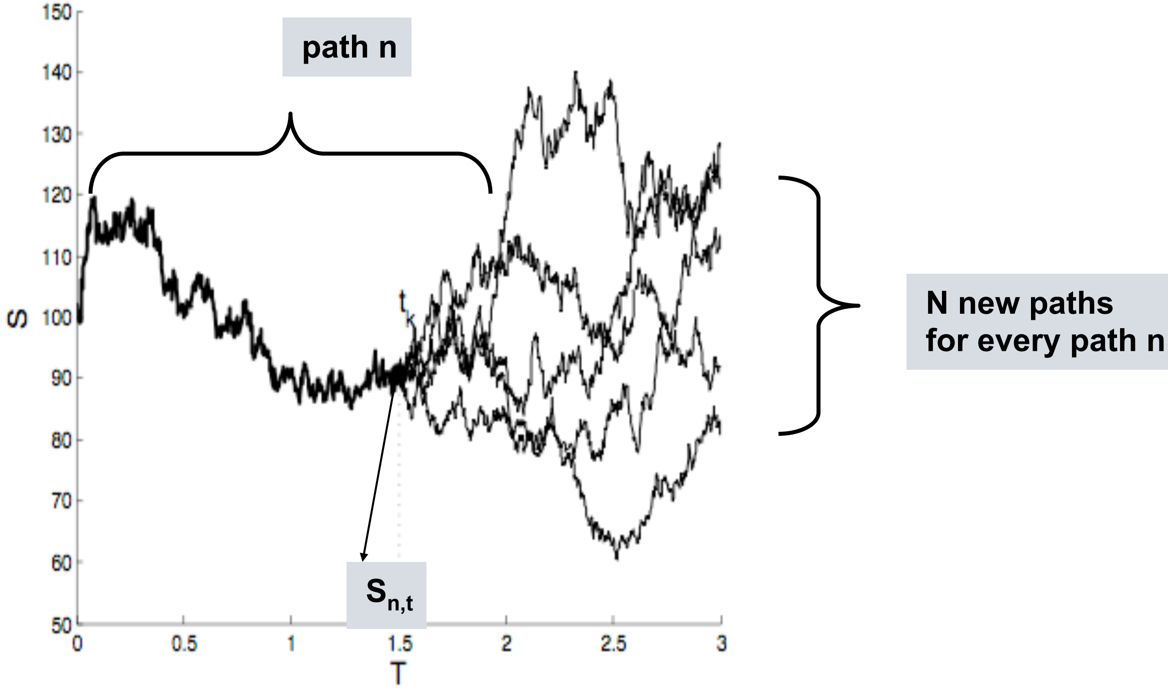
- 3 Random walks are simulated
- Suppose we know the exercise boundary above which the convertible will be converted into shares
  - **Path 1**
    - At t=0.69 year the share price path crosses the boundary where S = 121.68
    - The cash flow at this point is $121.68
    - Present Value = **119.19**
  - **Path 2**
    - No early conversion
    - Final conversion with S=107.10
    - Present Value = **97.88**
  - **Path 3**
    - No early conversion
    - Final conversion value < FV
    - S ends at 30.31.
    - The final cash flow = 100
    - Present Value = **91.39**



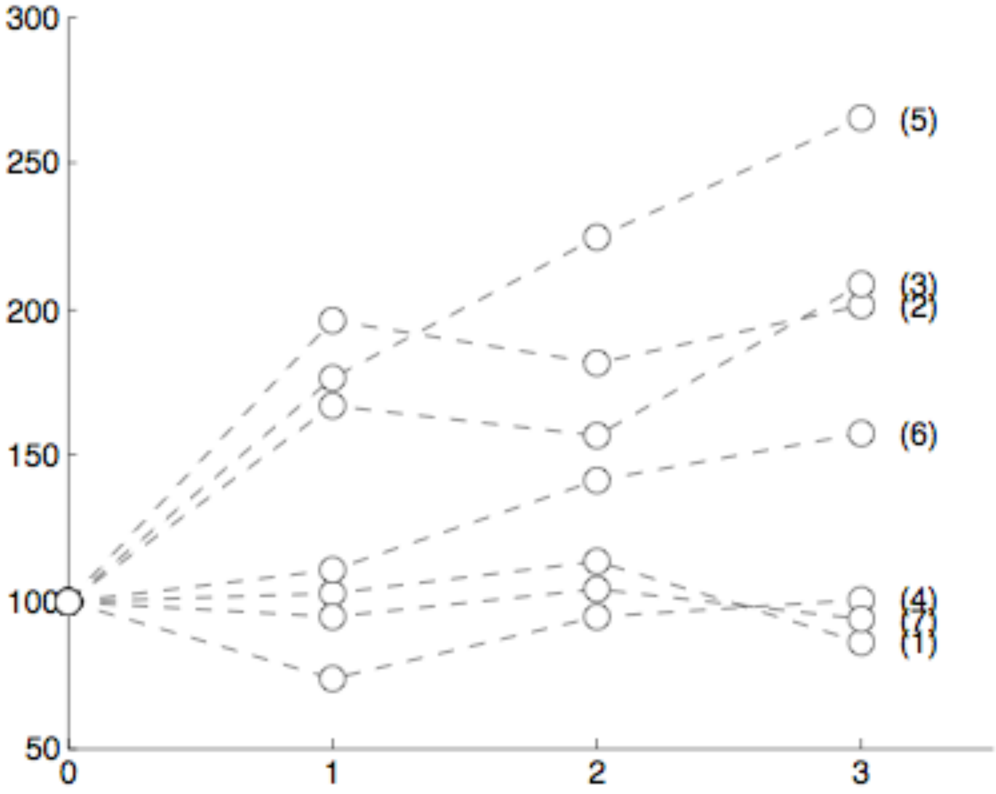
$$P = \frac{1}{3}(119.19 + 97.88 + 91.39) = 102.82$$

American Monte Carlo Technique Longstaff and Schwartz

- We have no knowledge of the exercise boundary
- At each point $S_{n,t}$ of path n where the convertible could be converted we calculate
  - **Exercise value** (P^*) = C_r x S
  - **Continuation Value** (P^c)
    This continuation value is the value of a convertible bond starting at $S_{n,t}$.
- Explosive in calculation time, if we apply Monte Carlo again.



American Monte Carlo Technique
Longstaff and Schwartz : calculation example



0	1	2	3
100.00	102.89	114.05	86.04
100.00	196.49	181.21	200.98
100.00	167.15	156.66	208.60
100.00	73.36	94.93	100.58
100.00	176.26	224.46	265.28
100.00	111.08	141.68	157.44
100.00	94.88	104.26	94.26

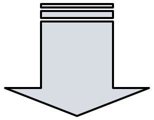
- 7 paths
- 2 intermediate conversion dates t=1 and t=2

t=3

American Monte Carlo Technique Longstaff and Schwartz

- Cash flows at the final maturity date determine the value of the convertible at the final date.
- 7 paths :
  - 5 conversions into shares
  - 2 redemptions at par
- These cash flows need to be rolled back to the previous point t=2.

3	
100.00	Redemption
200.98	Conversion
208.60	Conversion
100.58	Conversion
265.28	Conversion
157.44	Conversion
100.00	Redemption



t=2

t=3

American Monte Carlo Technique
Longstaff and Schwartz

CashFlow Matrix

Path	1	2	3
1			100.00
2			200.98
3			208.60
4			100.58
5			265.28
6			157.44
7			100.00

t=2

American Monte Carlo Technique Longstaff and Schwartz

- In each of the 7 paths we need to check if the convertible should be converted.
- Conversion if
  Exercise Value > Continuation Value
  $P^*>P^c$
- **Step 1**
  Determine continuation values where conversion is not possible
- **Step 2**
  Obtain other continuation values through a regression
- **Step 3**
  Calculate the value of the convertible P in all paths and move on...

Path	S	P^*	P^c	P
1	114.05	114.05		
2	181.21	181.21		
3	156.66	156.66		
4	94.93	94.93		
5	224.46	224.46		
6	141.68	141.68		
7	104.26	104.26		

t=2

American Monte Carlo Technique Longstaff and Schwartz

- **Step 1**
  - Path 4 corresponds to a share price of 94.93 for t=2.
  - The convertible will not be exercised at this point
  - The value of the convertible (P) in this point is equal to the present value of the convertible along the same path in the previous node at t=3
  - This value
    - P = 94.91 x exp(-0.03)
    - P = 97.61

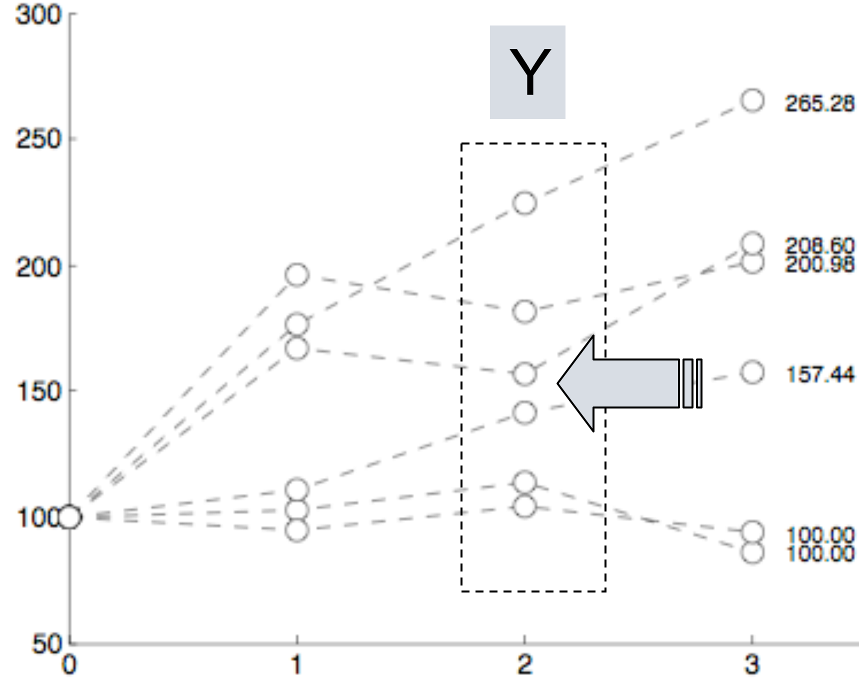
Path	S	P^*	P^c	P
1	114.05	114.05		
2	181.21	181.21		
3	156.66	156.66		
4	94.93	94.93	97.61	97.61
5	224.46	224.46		
6	141.68	141.68		
7	104.26	104.26		

t=2

American Monte Carlo Technique Longstaff and Schwartz

- **Step 2**
  - In the remaining points t∈{1,2,3,5,6,7}, the continuation value needs to be calculated.
  - A first step is to calculate the present value **Y** of cash flows paid out at t=3 in the nodes t=2.
  - The values of Y are **not** the continuation values

Path	S	Y
1	114.05	97.04
2	181.21	195.04
3	156.66	202.43
5	224.46	257.44
6	141.68	152.79
7	104.26	97.04



t=2

American Monte Carlo Technique Longstaff and Schwartz

- **Step 2**
  - Longstaff and Schwartz have proven that the continuation value $\mathbf{P^c}$ can be written as :
  - $\mathbf{P^c=a+bS+cS^2+dS^3}$ (*)
  - The coefficients a,b,c and d are obtained using a regression against **Y**

Coefficient	Value
a	-474.8802
b	9.0848
c	-0.0436
d	0.0001

  - Using the results of this regression we can calculate the values of the $\mathbf{P^c}$ in each of the paths where conversion could be possible. For example in the first path where S=114.05, we find $\mathbf{P^C}$ **= 110.66**
  - This calculation is to be done for path 1,2,3,5,6 and 7

Path	S	P*	Pc	P
1	114.05	114.05	**110.66**	
2	181.21	181.21	**207.08**	
3	156.66	156.66	**180.31**	
4	94.93	94.93	97.61	97.61
5	224.46	224.46	**255.87**	
6	141.68	141.68	**160.47**	
7	104.26	104.26	**87.39**	

(*) This is only one of the possible choices of basis functions.

t=2

American Monte Carlo Technique Longstaff and Schwartz

- **Step 3**
  - Compare P^* and P^c and determine the value of the convertible P in each of the paths at time t=2
  - In t=2, there are two paths where the convertible will be converted into shares
  - This conversion will generate an intermediate cash flow at t=2

Path	S	P*	Pc	P	Action
1	114.05	114.05	110.66	114.05	Convert
2	181.21	181.21	207.08	207.08	
3	156.66	156.66	180.31	180.31	
4	94.93	94.93	97.61	97.61	
5	224.46	224.46	255.87	255.87	
6	141.68	141.68	160.47	160.47	
7	104.26	104.26	87.39	104.26	Convert

t=2

American Monte Carlo
Longstaff and Schwartz

CashFlow Matrix

Path	1	2	3
1		114.05	
2			200.98
3			208.60
4		97.61	
5			265.28
6			157.44
7		104.26	

American Monte Carlo Longstaff and Schwartz

- **Step 1**
  - There are in t=1, two points where conversion is suboptimal : path 4 and 7.
  - The continuation value (**P**^{c}) in these nodes is equal to the present value of the subsequent cash flows:
    - 94.73 = exp(-0.03) x 97.61
    - 101.18 = exp(-0.03) x 104.26
- **Step 2**
  - Calculate in the nodes 1,2,3,5 and 6 the present value if the subsequent cash flows : **Y**

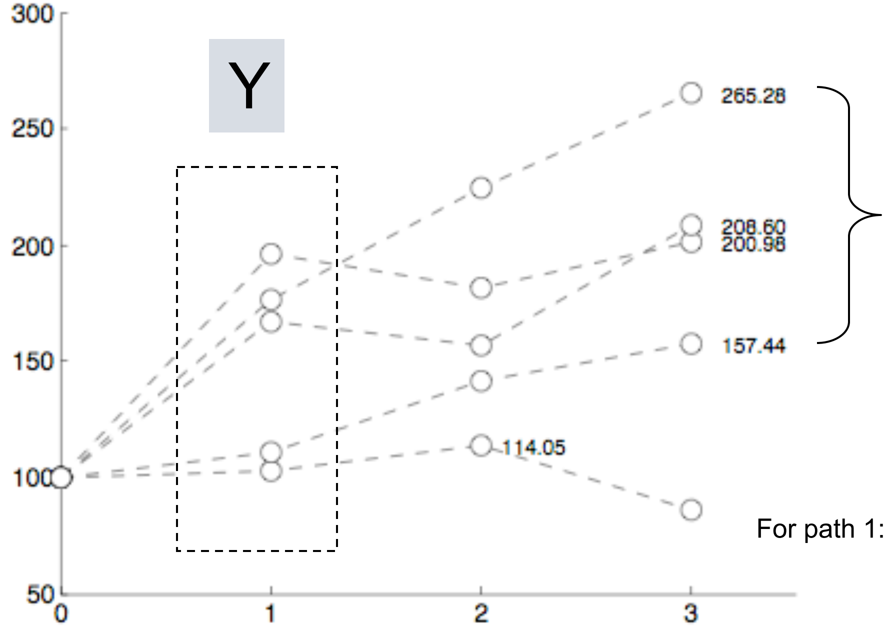
Path	S	P*	Pc	P
1	102.89	102.89		
2	196.49	196.49		
3	167.15	167.15		
4	73.36	73.36	94.73	94.73
5	176.26	176.26		
6	111.08	111.08		
7	94.88	94.88	101.18	101.18

t=1

American Monte Carlo
Longstaff and Schwartz

- **Step 2**
  Calculation of Y

Path	S	Y
1	102.89	110.68
2	196.49	189.26
3	167.15	196.44
5	176.26	249.81
6	111.08	148.26



Y= Present value of the cash flows paid out at maturity

For path 1: Y= Present value of the cash flow paid out at t=2

American Monte Carlo
Longstaff and Schwartz

- **Step 2**
  - Through a regression of Y vs [1 S S^2 S^3] the continuation value can be calculated:
  - **$P^c=a+bS+cS^2+dS^3$**

Coefficient	Value
a	-54.9230
b	-0.7007
c	0.0375
d	-0.0001

  - Calculate the continuation values using the formula for path 1,2,3,5 and 6

Path	S	P*	Pc	P
1	102.89	102.89	117.71	
2	196.49	196.49	194.61	
3	167.15	167.15	222.78	
4	73.36	73.36	94.73	94.73
5	176.26	176.26	221.03	
6	111.08	111.08	138.33	
7	94.88	94.88	101.18	101.18

t=1

American Monte Carlo
Longstaff and Schwartz

- Step 3
  - Compare P* and Pc and determine the value of the convertible in every node of the paths at time t=1.

Path	S	P*	Pc	P	
1	102.89	102.89	117.71	117.71	
2	196.49	196.49	194.61	196.49	Convert
3	167.15	167.15	222.78	222.78	
4	73.36	73.36	94.73	94.73	
5	176.26	176.26	221.03	221.03	
6	111.08	111.08	138.33	138.33	
7	94.88	94.88	101.18	101.18	

  - There is only one node where the convertible will be converted into shares at t=2

t=1

American Monte Carlo
Longstaff and Schwartz

CashFlow Matrix

Path	1	2	3
1		114.05	
2	196.49		
3			208.60
4	94.73		
5			265.28
6			157.44
7	101.18		

t=0

American Monte Carlo
Longstaff and Schwartz

- This is the **final step**
- The value of the convertible is equal to the expected present value of the converts in t=1
- Price of the convertible = **152.16**

- **More Precision:**
  - Increase nbr of paths
  - Increase nbr of points where the early conversion is checked.

Bio

Jan De Spiegeleer (Geneva, Switzerland) is head of risk management at Jabre Capital Partners, a Geneva-based hedge fund. He earned an extensive knowledge of derivatives pricing, hedging and trading while working for KBC Financial Products in London, where he was managing director of the equity derivatives desk. He also ran his own market neutral statistical arbitrage hedge fund (EQM Europe) after founding Erasmus capital in 2004. Prior to his financial career, Jan has spend ten years in the Belgian Army during which he also served in Iraq.

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